

# Exercises

$$1. x^5 = 5x^4$$

$$2. x^{\sqrt{3}} = \sqrt{3} x^{\sqrt{3}-1}$$

$$3. \frac{1}{t^3} = t^{-3} = -3t^{-4} = -\frac{3}{t^4}$$

$$4. \frac{4}{u^4} = 4u^{-4} = -16u^{-5} = -\frac{16}{u^5}$$

$$5. \frac{1}{5v^5} = \frac{1}{5} v^{-5} = -v^{-6} = -\frac{1}{v^6}$$

$$6. \frac{x^3}{7} = \frac{1}{7} x^3 = x^6$$

$$7. \frac{1}{3\sqrt{x^2}} = x^{-\frac{2}{3}} = -\frac{2}{3} x^{-\frac{5}{3}} = -\frac{2}{3x^{\frac{5}{3}}} = -\frac{2}{3x^{\frac{5}{3}}\sqrt{x^2}}$$

$$8. 2x - x^3 = 2 - 3x^2$$

$$9. 4x^3 - 3x^2 + 7 = 12x^2 - 6x$$

$$10. 5 - 2x^2 + x^4 = -4x + 4x^3$$

$$11. 3x^4 - 7x^3 + 5x^2 + 8 = 12x^3 - 21x^2 + 10x$$

$$12. 4x^3 + 2 + \frac{1}{x} = 12x^2 - x^{-2} = 12x^2 - \frac{1}{x^2} = \frac{12x^4 - 1}{x^2}$$

$$13. 3u^2 + \frac{3}{v^2} = 3u^2 + 3v^{-2} = 6u - 6v^{-3} = 6u - \frac{6}{v^3} = \frac{6v^4 - 6}{v^3}$$

$$14. \frac{x^6}{6} + \frac{6}{x^6} = \frac{1}{6} x^6 + 6x^{-6} = x^5 - 36x^{-7} = x^5 - \frac{36}{x^7} = \frac{x^{12} - 36}{x^7}$$

$$15. x^{12} + \frac{1}{x^{0.6}} = x^{6/5} + x^{-\frac{3}{5}} = \frac{6}{5} x^{\frac{1}{5}} - \frac{3}{5} x^{-\frac{8}{5}} = \frac{6}{5} \sqrt[5]{x} - \frac{3}{5x^{\frac{8}{5}}}$$

$$16. 2\sqrt{x} + \frac{2}{\sqrt{x}} = 2x^{\frac{1}{2}} + 2x^{-\frac{1}{2}} = x^{\frac{1}{2}} + x^{-\frac{3}{2}} = \frac{1}{\sqrt{x}} - \frac{1}{x\sqrt{x}}$$

$$17. x^3 + \frac{1}{x^4} + 7x + \frac{7}{x} + 7 = 7x^6 - 7x^{-8} + 7 - 7x^{-2} = 7x^6 - \frac{7}{x^8} + 7 - \frac{7}{x^2}$$

$$19. 2\sqrt{x^3} + \frac{2}{\sqrt{x^3}} = 2x^{\frac{3}{2}} + 2x^{-\frac{3}{2}} = 3x^{\frac{1}{2}} - 3x^{-\frac{3}{2}} = 3\sqrt{x} - \frac{3}{x^{\frac{3}{2}}\sqrt{x}}$$

$$20. 2\sqrt{t} - \frac{3}{\sqrt{t}} = 2t^{\frac{1}{2}} - 3t^{-\frac{1}{2}} = t^{-\frac{1}{2}} - t^{-\frac{3}{2}} = \frac{1}{\sqrt{t}} - \frac{1}{t^{\frac{3}{2}}\sqrt{t}}$$

$$21. 2x^{\frac{3}{2}} + 4x^{\frac{5}{2}} = 3x^{\frac{1}{2}} + 10x^{\frac{3}{2}} = 3\sqrt{x} + 10x^{\frac{3}{2}}\sqrt{x^2}$$

$$22. \sqrt[3]{x} - \frac{1}{\sqrt[3]{x}} = x^{\frac{1}{3}} - x^{-\frac{1}{3}} = \frac{1}{3} x^{-\frac{2}{3}} + \frac{1}{3} x^{-\frac{4}{3}} = \frac{1}{3\sqrt[3]{x^2}} + \frac{1}{3x^{\frac{4}{3}}\sqrt{x}}$$

$$23. 3x^4 + (2x-1)^2 = 12x^3 + [2(2x-1) \cdot (2)] = 12x^3 + 8x - 4$$

$$24. (4-2)(24-3) = (4-2)(2) + (24-3)(1) = 24 - 4 + 24 - 3 = 44 - 7$$

$$25. (x-7)(24-9) = (x-7)(2) + (24-9)(1) = 2x - 14 + 24 - 9 = 4x - 7$$

$$1. y = (x+1)(x^3+3) = y' = (x+1)(3x^2) + (x^3+3)(1) = 4x^3 + 3x^2 + 3$$

$$2. y = (x^3+6x^2)(x^2-1) = y' = (x^3+6x^2)(2x) + (x^2-1)(3x^2+12x)$$

$$3. y = (7x+1)(2-3x) = y' = (7x+1)(-3) + (2-3x)(7)$$

$$4. y = (x^2+7x)(x^2+3x+1) = y' = (x^2+7x)(2x+3) + (x^2+3x+1)(2x+7)$$

$$5. f(x) = (x^2-5x+1)(2x+3) = (x^2-5x+1)(2) + (2x+3)(2x-5)$$

$$6. f(x) = (x^2+1)(x+1)^2 = [(x^2+1) \cdot 2(x+1) \cdot 1] + (x+1)^2(2x)$$

$$f'(x) = (x^2+1)(2x+2) + (x+1)^2(2x)$$

$$7. f(x) = (3x+7)(x-1)^2 = f'(x) = (3x+7) \cdot 2(x-1)(1) + (x-1)^2(3)$$

$$f'(x) = (3x+7)(2x-2) + (x-1)(3)$$

$$8. y = (t^2+1)\left(t - \frac{1}{t}\right) = y' = (t^2+1)\left(1+t^{-2}\right) + \left(t - \frac{1}{t}\right)(2t)$$

$$y' = (t^2+1)\left(1 + \frac{1}{t^2}\right) + \left(t - \frac{1}{t}\right)(2t)$$

$$9. f(x) = \left(y + \frac{3}{y}\right)(y^2-5) = f'(x) = \left(y + 3y^{-2}\right)(2y) + (y^2-5)(1-3y^{-3})$$

$$f'(x) = \left(y + \frac{3}{y}\right)(2y) + (y^2-5)\left(1 - \frac{3}{y^3}\right)$$

$$10. f(t) = \left(t + \frac{1}{t}\right)\left(5t^2 - \frac{1}{t^2}\right) = f'(t) = \left(t + \frac{1}{t}\right)(10t + 2t^{-2}) + \left(5t^2 - \frac{1}{t^2}\right)\left(1 - t^{-2}\right)$$

$$f'(t) = \left(t + \frac{1}{t}\right)\left(10t + \frac{2}{t^3}\right) + \left(5t^2 - \frac{1}{t^2}\right)\left(1 - \frac{1}{t^2}\right)$$

$$11. f(x) = (x^2+1)(3x-1)(2x-3) = f'(x) = u^2 \cdot v \cdot w + u \cdot v^2 \cdot w + u \cdot v \cdot w^2$$

$$f'(x) = (2x)(3x-1)(2x-3) + (x^2+1)(3)(2x-3) + (x^2+1)(3x-1)(2)$$

$$12. f(x) = (2x+1)(3x^2+1)(x+7) = f'(x) = (2)(3x^2+1)(x+7) + 6x(2x+1)(x+7) + 3x^2(2x+1)(3x^2+5)$$

$$19. y = \frac{3}{2x+7} = y' = \frac{(2x+7)(0) - 3(2)}{(2x+7)^2} = \frac{-6}{(2x+7)^2}$$

$$20. f(t) = \frac{5t}{2-3t} = f'(t) = \frac{(2-3t)(5) - (5t)(-3)}{(2-3t)^2} = \frac{10}{(2-3t)^2}$$

$$21. y = \frac{u}{u+1} = y' = \frac{(u+1)(1) - u(1)}{(u+1)^2} = \frac{1}{(u+1)^2}$$

$$22. f(x) = \frac{x+1}{x+3} = f'(x) = \frac{(x+3)(1) - (x+1)(1)}{(x+3)^2} = \frac{2}{(x+3)^2}$$

$$23. f(x) = \frac{x+2}{x+1} = f'(x) = \frac{(x+1)(1) - (x+2)(1)}{(x+1)^2} = -\frac{1}{(x+1)^2}$$

$$24. g(x) = \frac{3-x}{x^2-3} = g'(x) = \frac{(x^2-3)(-1) - (3-x)(2x)}{(x^2-3)^2} = \frac{x^2-6x+3}{(x^2-3)^2}$$

$$25. y = \frac{t^2-7t}{t-5} = y' = \frac{(t-5)(2t-7) - (t^2-7t)(1)}{(t-5)^2} = \frac{t^2-10t+35}{(t-5)^2}$$

$$26. y = \frac{u^2-u+1}{u^2+u+1} = y' = \frac{(u^2+u+1)(2u-1) - (u^2-u+1)(2u+1)}{(u^2+u+1)^2}$$

$$7. h(t) = \sqrt{t^2 + a^2} = (t^2 + a^2)^{1/2} = h'(t) = \frac{1}{2} (t^2 + a^2)^{-1/2} \cdot (2t + 2a)$$

$$h'(t) = (2t + 2a) \left( \frac{1}{2\sqrt{t^2 + a^2}} \right)$$

$$8. F(x) = \sqrt[3]{x^3 + 3x} = F'(x) = \frac{1}{3} (x^3 + 3x)^{-2/3} \cdot (3x^2 + 3)$$

$$9. x = \frac{1}{\sqrt[3]{t^3 + 1}} = x' = \frac{1}{(t^3 + 1)^{4/3}} = (t^3 + 1)^{-4/3} = -\frac{1}{3} (t^3 + 1)^{-7/3} \cdot (3t^2)$$

$$= -\frac{3t^2}{3} (t^3 + 1)^{-7/3} = \frac{-t^2}{(t^3 + 1)^{7/3}}$$

$$10. y = \left(t + \frac{1}{t}\right)^{10} = y' = 10 \left(t + \frac{1}{t}\right)^9 \cdot (1 - t^{-2})$$

$$11. y = \left(t^2 + \frac{1}{t^2}\right)^5 = y' = 5 \left(t^2 + \frac{1}{t^2}\right)^4 \cdot (2t - 2t^{-3})$$

$$12. y = \frac{1}{\sqrt{u^2 + 9}} = y' = (u^2 + 9)^{-1/2} = -\frac{1}{2} (u^2 + 9)^{-3/2} \cdot (2u)$$

$$= -\frac{2u}{2} (u^2 + 9)^{-3/2} = -\frac{u}{(u^2 + 9)^{3/2}}$$

$$13. y = (x^2 + 1)^{0.6} = y' = (x^2 + 1)^{-1/5} = \frac{3}{5} (x^2 + 1)^{-2/5} \cdot (2x) = y' = \frac{6x}{5(x^2 + 1)^{2/5}}$$

$$14. y = \sqrt{x + \frac{1}{x^2}} = y' = \left(x + \frac{1}{x^2}\right)^{-1/2} = \frac{1}{2} \left(x + \frac{1}{x^2}\right)^{-3/2} \cdot (1 - 2x^{-3})$$

$$15. v^3 \sqrt{t^3 - \frac{1}{t^3}} = v' = \left(t^3 - \frac{1}{t^3}\right)^{1/3} = \frac{1}{3} \left(t^3 - \frac{1}{t^3}\right)^{-2/3} \cdot (3t^2 + 3t^{-4})$$

$$16. y = \sqrt{1 + x \ln 2} = y' = \frac{1}{2} (1 + x \ln 2)^{-1/2} \cdot (\ln 2) = y' = \frac{\ln 2}{2(1 + x \ln 2)^{1/2}}$$

$$17. f(x) = \frac{\sqrt{x^2 + 1}}{\sqrt[3]{x^2 + 1}} = f'(x) = (x^2 + 1)^{1/3} \cdot \frac{1}{2} (x^2 + 1)^{-1/2} (2x) - (x^2 + 1)^{1/2} \cdot \frac{2}{3} (x^2 + 1)^{-4/3} (2x)$$

$$18. g(x) = (x^4 + 16)^{14} = g'(x) = 14(x^4 + 16)^{13} \cdot (4x^3)$$

$$19. G(u) = (u^2 + 2)^3 (2u + 1) = G'(u) = (u^2 + 2)^3 (2) + (2u + 1) \cdot 3(u^2 + 2)^2 (2u)$$

$$20. H(y) = (2y^2 + 3)^6 (5y + 5) = H'(y) = (2y^2 + 3)^6 (5) + (5y + 5) \cdot 6(2y^2 + 3)^5 (4y)$$

$$21. y = \left(\frac{3x + 2}{x - 1}\right)^7 = y' = \frac{(x - 1)^7 \cdot 7(3x + 2)^6 (3) - (3x + 2)^7 \cdot 7(x - 1)^6}{((x - 1)^1)^2}$$

$$28. y = \left( \frac{t}{t+1} \right)^6 = y^2 = \frac{t^6}{(t+1)^6} = \frac{(t+1)^6 \cdot 6t^5 - t^6 \cdot 6(t+1)^5}{((t+1)^6)^2}$$

$$29. y = \left( \frac{v^2+1}{v+1} \right)^3 = y^3 = \frac{(v+1)^3 \cdot 3(v^2+1)^2(2v) - (v^2+1)^3 \cdot 3(v+1)^2}{((v+1)^3)^2}$$

$$30. y = \sqrt{\frac{3x+7}{5+2x}} = \frac{(3x+7)^{1/2}}{(5+2x)^{1/2}} = y^2 = \frac{(5+2x)^{1/2} \cdot \frac{1}{2}(3x+7)^{-1/2} \cdot 3 - (3x+7)^{1/2} \cdot \frac{1}{2}(5+2x)^{-1/2} \cdot 2}{((5+2x)^{1/2})^2}$$